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17MATDIP31

Third Semester B.E. Degree Examination, Dec.2018/Jan.2019
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Prove that $(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n=2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$ (08 Marks)

b. Express $\sqrt{3}+i$ in the polar form and hence find its modulus and amplitude. (06 Marks)

c. Find the sine of the angle between vectors $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}-3\hat{j}+2\hat{k}$ (06 Marks)

OR

2 a. Express $\frac{3+4i}{3-4i}$ in the form x + iy. (08 Marks)

b. If the vector $2\hat{i}+\lambda\hat{j}+\hat{k}=0$ and $4\hat{i}-2\hat{j}-2\hat{k}$ are perpendicular to each other, find λ. (06 Marks)

c. Find λ, such that the vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}+2\hat{j}-3\hat{k}$, $3\hat{i}+\lambda\hat{j}+5\hat{k}$ are coplanar. (06 Marks)

Module-2

3 a. If $y=e^{a\sin^{-1}x}$, prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0$ (08 Marks)

b. With usual notations, prove that $\tan\phi=r\frac{d\theta}{dr}$. (06 Marks)

c. If $u=\log_e\frac{x^3+y^3}{x^2+y^2}$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=1$. (06 Marks)

OR

4 a. Using Maclaurin's series, expand $\tan x$ upto the term containing x^5. (08 Marks)

b. Find the pedal equation of $r=a(1-\cos\theta)$. (06 Marks)

c. If $u=x+3y^2-z^3$, $v=4x^2yz$ and $w=2z^2-xy$, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1, –1, 0). (06 Marks)

Module-3

5 a. Obtain a reduction formula for $\int_0^{\pi/2}\cos^n x\,dx$, (n > 0). (08 Marks)

b. Evaluate $\int_0^a\frac{x^7}{\sqrt{a^2-x^2}}dx$ (06 Marks)

c. Evaluate $\int_1^2\int_1^3 xy^2\,dx\,dy$ (06 Marks)

OR

6 a. Obtain a reduction formula for $\int_0^{\pi/2} \sin^n x\,dx$, $(n > 0)$. **(08 Marks)**

b. Evaluate $\int_0^{2a} x^2\sqrt{2ax - x^2}\,dx$ **(06 Marks)**

c. Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x + y + z)\,dx\,dy\,dz$ **(06 Marks)**

Module-4

7 a. A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$ and $z = 3t - 5$, where 't' is the time. Find its velocity and acceleration vectors and also magnitude of velocity and acceleration at $t = 1$. **(08 Marks)**

b. In which direction of the directional derivative of x^2yz^3 is maximum at (2, 1, –1) and find the magnitude of this maximum. **(06 Marks)**

c. Show that $\vec{F} = (y + z)\hat{i} + (x + z)\hat{j} + (x + y)\hat{k}$ is irrotational. **(06 Marks)**

OR

8 a. If $\phi = xy^2z^3 - x^3y^2z$, find $\nabla\phi$ and $|\nabla\phi|$ at (1, –1, 1). **(08 Marks)**

b. If $\vec{F} = (x + y + 1)\hat{i} + \hat{j} - (x + y)\hat{k}$, show that $\vec{F} \cdot \text{Curl}\,\vec{F} = 0$. **(06 Marks)**

c. If $x = t^2 + 1$, $y = 4t - 3$, $z = 2t^2 - 6t$ represents the parametric equation of a curve, find the angle between the tangents at $t = 1$ and $t = 2$. **(06 Marks)**

Module-5

9 a. Solve : $\left(x\tan\frac{y}{x} - \frac{y}{x}\sec^2\frac{y}{x}\right)dx = x\sec^2\frac{y}{x}dy$ **(08 Marks)**

b. Solve : $xy(1 + xy^2)\frac{dy}{dx} = 1$ **(06 Marks)**

c. Solve : $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$ **(06 Marks)**

OR

10 a. Solve : $(3y + 2x + 4)dx - (4x + 6y + 5)dy = 0$ **(08 Marks)**

b. Solve : $(1 + y^2)dx = (\tan^{-1}y - x)dy$ **(06 Marks)**

c. Solve : $(y\log y)dx + (x - \log y)dy = 0$. **(06 Marks)**

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